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On the Scattering of Gravitons on Two Parallel D-Branes

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ABSTRACT

I discuss the scattering of a graviton (or a dilaton, or an anti-symmetric tensor) on two parallel static Dp-branes. The graviton belongs to a type II string in 10D.

1. Introduction

Many interesting results have been recently obtained from the computation of string scattering amplitudes in presence of Dp-branes, for a review see for example ref. [1] and references therein. Most of the computations so far appeared in the literature concern the case of the scattering of a particle on a single Dp-brane. The external particle belongs to a closed string theory, in 10D a type II string, which interacts with the open Dirichelet string describing the D-brane. Thus from a perturbative string point of view the scattering of, for example, a graviton on a Dp-brane is described by the amplitude where the two vertex operators of the incoming and outgoing graviton are inserted in the bulk of a disk.

The next case in perturbation theory is the one in which there are two parallel Dp-branes and an open string stretching between them. At first one assumes that the Dp-branes are fixed in space, or in other words, one completely disregards their dynamics, even their recoil. The amplitude describing this scattering is given by the insertion of the two vertex operators in the bulk of a cylinder which is the world-sheet spanned by the open string. More generally, a scattering of a graviton on n parallel Dp-branes is described by a "$n-1$ loop" amplitude in open-string theory with the two graviton vertex operators inserted in the bulk.

In the case of two parallel Dp-branes, one has to compute amplitudes with closed string vertex operators inserted on the cylinder (see for example ref. [2]). The only novelties in the computation are the modifications due to the fact that the boundary conditions of the open string are Neumann in $p+1$ directions, and Dirichelet in the remaining directions.

In this letter I will describe this computation in the case of the scattering of a graviton (or a dilaton or an anti-symmetric tensor) belonging to a D=10 type II string theory, on two parallel Dp-branes.

Before getting to the computation, it is worth to discuss some issues. As already said, the Dp-branes are assumed to be fixed in space and indeed in the computation I will completely ignore their dynamics, even their recoil. Whereas on the disc, i.e. for the scattering on a single Dp-brane, this did not lead to any pathology, on the cylinder a priori, the non-conservation of the momentum in the direction orthogonal to the branes, could lead to some "inconsistencies" in the final result.

Indeed, in the explicit computation, the external particles are on-shell but the momentum is not conserved since part of it is absorbed by the Dp-branes. This situation can be thought of as if some particles were off-shell, and indeed Dirichelet boundary condition were introduced some time ago as a way of computing off-shell string scattering amplitudes [3,4]. But it is very well-known how subtle is the process of "going off-shell" in a string scattering amplitude since it also means violating the 2d conformal invariance on which the perturbative formulation of the scattering amplitude is based.

For example, in the presence of the Dp-brane and since the momentum is not fully conserved, one could be worried that the final expression of the amplitude could cease to be independent from the insertion points of the Picture Changing Operators, could be not gauge invariant and there could exist some "conformal anomalies" [4]. This implies that the computation could be plagued by ambiguities and the result must then be handled with care.

Of course, the best approach would be to take in consideration also the dynamics of the D-branes. But since this is not yet possible, one can reverse the argument and from a detailed study of the properties of this and similar amplitudes, try to learn something on the perturbative dynamics (at least of the recoil) of the D-branes [5,6].

2. Generalities on the Scattering

Consider two parallel Dp-branes at distance ΔY_μ and an open string which connects them. As usual the open string fields X_μ, ψ_μ have Neumann boundary conditions for $\mu = 0, \ldots, p$ and Dirichelet boundary conditions for $\mu = p+1, \ldots, 9$. The scattering of a graviton (or a Kalb-Ramond tensor or a dilaton) of a 10D type II string on the two parallel Dp-branes, i.e. on the open string which connects them, is then described by an amplitude on a cylinder with the two closed string vertex operators, describing the incoming and outgoing particles, inserted in the bulk.

I work in the usual string formalism following mostly ref. [2], my conventions for the prime form and the theta functions are as in refs. [7,8,9]. For what concerns the Dp-brane, I follow mostly the conventions and notations of ref. [10]. I use the NSR formalism and I am fully covariant (all ghosts and superghosts). This formalism is the most practical for extending these computations also to scattering of space-time fermions and other particles.

To fix the notation I first recall the form of the partition function.

2.1 *The Partition function*

The partition function (see [11]) for a Dp-brane is

$$
\begin{aligned}
Z_p \;&= N_1^p \int_0^\infty \mathrm{d}\Im m\tau \;\times\; \frac{(k)^{-26/24}(\eta(\tau))^2}{\Im m\tau} \;\times \\
&\quad e^{-(\Delta Y)^2\Im m\tau/4\pi}(k)^{10/24}(\eta(\tau))^{-10}(2\pi\Im m\tau)^{-(p+1)/2} \;\times \\
&\quad \sum_{\alpha,\beta} C^\alpha_\beta \left(\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)\right)^5 (k)^{5/24}(\eta(\tau))^{-5} \;\times\; \left(\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)\right)^{-1}(k)^{1/2-1/24}\eta(\tau)
\end{aligned}
\tag{1}
$$

where I divided the contribution of bc-ghosts, bosonic coordinates, fermionic coordinates and $\beta\gamma$-superghosts respectively. In this formula $k = \exp(2\pi i\tau)$, $N_1^p = (4\pi\alpha')^{-(p+1)/2}$ is the overall normalization, the sum is over only the even spin-structures (the odd spin-structure does not contribute) of the open string where $\alpha = 0$ is Ramond (R) and $\alpha = 1/2$ is Neveu-Schwartz (NS), and $C^\alpha_\beta = \frac{1}{2}\exp[2\pi i(\alpha+\beta)]$. Moreover, on the cylinder $\Re e\tau = 0$.

The partition function vanishes by summing over the spin-structures.

3. The two-point amplitude

The vertex operator for a graviton (or a anti-symmetric tensor or a dilaton) in the zero super-ghost picture is

$$
\begin{aligned}
V^{Dp}(\bar z, z; k, \zeta) \;&=\; \zeta^{\rho_1\nu_2}\frac{\kappa}{\pi}\left[\bar\partial \bar X_{\rho_1}(z,\bar z) - ik\cdot\bar\psi(\bar z)\bar\psi_{\rho_1}(\bar z)\right] \;\times \\
&\qquad \left[\partial X_{\nu_2}(z,\bar z) - ik\cdot\psi(z)\psi_{\nu_2}(z)\right]e^{ik\cdot X(z,\bar z)}
\end{aligned}
\tag{2}
$$

where $k^\mu = \sqrt{\alpha'/2}\,p^\mu$, $k^2 = 0$, and the polarization tensor satisfies $k\cdot\zeta = \zeta\cdot k = 0$. The amplitude is given by

$$
\begin{aligned}
T^{(Dp)}(k_1,\zeta_1;k_2,\zeta_2) \;&=\; \int_0^\infty \frac{\mathrm{d}\Im m\tau}{\Im m\tau}\int \mathrm{d}^2 z_1 \mathrm{d}^2 z_2 \\
&\quad (8\pi^2\alpha'\Im m\tau)^{-(p+1)/2}(\eta(\tau))^{-12}e^{-(\Delta Y)^2\Im m\tau/4\pi} \\
&\quad \sum_{\alpha,\beta} C^\alpha_\beta \left(\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)\right)^4 \langle V^{Dp}(\bar z_1, z_1; k_1,\zeta_1)V^{Dp}(\bar z_2, z_2; k_2,\zeta_2)\rangle \;.
\end{aligned}
\tag{3}
$$

In my conventions, I extract from all correlators the contribution of the partition function so that the amplitude is given by the partition function times the correlator of the vertex operators. I am using vertex operators in the zero-superghost picture, so that the contribution to the amplitude of the superghosts is exactly the same as in the partition function and there is no need for the insertion of Picture Changing Operators.

I am not making any assumptions on the polarizations so that the result holds for gravitons, Kalb-Ramond tensors and dilatons.

To proceed in the computation of eq. (3) one notices immediately that the odd spin-structure does not contribute, since to have a contribution from the odd spin-structure one needs to have at least ten world-sheet fermions. [1] Then the sum is restricted to the even spin-structures. By the Riemann (or abstruse) identity, only terms with eight, or more, world-sheet fermions give a non vanishing contribution after the sum over the spin-structures. Thus *effectively* I can use the following vertex operator [2]

$$V^{Dp,eff}(\bar{z},z;k,\zeta) \;=\; -\zeta^{\rho_1\nu_2}\frac{\kappa}{\pi}\,k\cdot\bar{\psi}(\bar{z})\bar{\psi}_{\rho_1}(\bar{z})\;k\cdot\psi(z)\psi_{\nu_2}(z)e^{ik\cdot X(z,\bar{z})}\;. \tag{4}$$

As in ref. [10] I introduce the projectors V parallel to the Dp-brane and N orthogonal to the Dp-brane. Thus $g_{\mu\nu} = V_{\mu\nu} + N_{\mu\nu}$ and $D_{\mu\nu} = V_{\mu\nu} - N_{\mu\nu}$ with $g = (-1, +1, \ldots, +1)$. Notice also that $D_{\mu}{}^{\lambda}D_{\lambda\nu} = g_{\mu\nu}$. Momentum is conserved only in the directions parallel to the Dp-branes, i.e. $\sum_i (V\cdot k_i) = 0$, which for a two-point amplitude is equivalent to $k_1 + D\cdot k_1 + k_2 + D\cdot k_2 = 0$. Finally $t = -(k_1+k_2)^2 = -2k_1\cdot k_2$ is the momentum transferred to the Dp-brane, and $q^2 = k_1\cdot V\cdot k_1 = \frac{1}{2}k_1\cdot D\cdot k_1$ is the momentum flowing parallel to the world-volume of the brane.

With these notations, the effective vertex operator can be written as

$$V^{Dp,eff}(\bar{z},z;k,\zeta) \;= -\frac{\kappa}{\pi}(D\cdot\zeta)^{\nu_1\nu_2}(k\cdot D)^{\mu_1}k^{\mu_2}\psi_{\mu_1}(\bar{z})\psi_{\nu_1}(\bar{z})\psi_{\mu_2}(z)\psi_{\nu_2}(z)e^{ik\cdot X(z,\bar{z})}\,. \tag{5}$$

So what is left to do, is to compute the bosonic and fermionic correlators in eq. (3).

Similar bosonic correlators have appeared already in the literature, see for example refs. [4,12,13,14,15,16,17,18]. Since I am not integrating over the distance ΔY between the two Dp-branes, the result is

$$\begin{aligned}
\langle e^{ik_1\cdot X(z_1,\bar{z}_1)}e^{ik_2\cdot X(z_2,\bar{z}_2)}\rangle \;=\; & \prod_{i=1}^{2}\exp\left[ik_i\cdot N\cdot Y + \frac{1}{2\pi}k_i\cdot N\cdot\Delta Y\log\left(\frac{z_i}{\bar{z}_i}\right)\right]\;\times \\
& \exp\left[k_1\cdot k_2\left(g_X(z_1,z_2)+g_X(\bar{z}_1,\bar{z}_2)\right)\;+\right. \\
& \quad k_1\cdot D\cdot k_2\left(g_X(z_1,\bar{z}_2)+g_X(\bar{z}_1,z_2)\right)\;+ \\
& \quad \left. k_1\cdot D\cdot k_1\,g_X(\bar{z}_1,z_1) + k_2\cdot D\cdot k_2\,g_X(\bar{z}_2,z_2)\right] \\
= & \prod_{i=1}^{2}\exp\left[ik_i\cdot N\cdot Y + \frac{1}{2\pi}k_i\cdot N\cdot\Delta Y\log\left(\frac{z_i}{\bar{z}_i}\right)\right]\;\times
\end{aligned}$$

[1] This of course holds in 10 dimensions and for vertex operators in the zero super-ghost picture. In other dimensions or using different pictures, similar, but not identical, results hold.

[2] I stress the fact that this result holds only for this computation and in the zero super-ghost picture.

$$\exp\left[-\frac{t}{2}\left(g_X(z_1,z_2)-g_X(z_1,\bar{z}_2)-g_X(\bar{z}_1,z_2)+g_X(\bar{z}_1,\bar{z}_2)\right)\;+\right.$$
$$\left.2q^2\left(g_X(\bar{z}_1,z_1)+g_X(\bar{z}_2,z_2)-g_X(z_1,\bar{z}_2)-g_X(\bar{z}_1,z_2)\right)\right] \tag{6}$$

where

$$g_X(z_1,z_2) \;=\; g_{osc}(z_1,z_2)+g_{zero}(z_1,\bar{z}_1;z_2,\bar{z}_2) \tag{7}$$

and

$$\begin{aligned} g_{osc}(z_1,z_2) &= \log E(z_1,z_2) \\ g_{zero}(z_1,\bar{z}_1;z_2,\bar{z}_2) &= -\frac{1}{16\pi\Im m\tau}(\log z_1+\log\bar{z}_1-\log z_2-\log\bar{z}_2)^2 \end{aligned} \tag{8}$$

(notice that g_X is a function of $(z_1,\bar{z}_1;z_2,\bar{z}_2)$).

The correlator of the world-sheet fermions does not depend on the position of the Dp-branes, thus the full dependence on Y and ΔY is given by

$$e^{-(\Delta Y)^2\Im m\tau/4\pi}\prod_{i=1}^{2}\exp\left[ik_i\cdot N\cdot Y+\frac{1}{2\pi}k_i\cdot N\cdot\Delta Y\log\left(\frac{z_i}{\bar{z}_i}\right)\right] \tag{9}$$

It is interesting to see what one obtains by integrating out the dependence on Y. In physical terms this means summing up all possible positions of the Dp-branes and this should establish the full momentum conservation.

Indeed, following for example ref. [16], one can introduce a momentum p conjugate to Y and a momentum $\hat{p}$ conjugate to ΔY and make a Fourier transform. The integration over Y gives a $\delta(p+N\cdot k_1+N\cdot k_2)$, which together with the momentum conservation in the directions parallel to the Dp-branes gives $\delta(p+k_1+k_2)$. Thus $p^2=-t$ is the overall momentum transferred to the system of the Dp-branes from the scattering particles. $\hat{p}$ is then the momentum of one of the two branes. As observed in ref. [16], if one chooses to keep one of the two branes fixed in space so that $\hat{p}=0$ and all the momentum is transferred to the other, equation (6) simplifies considerably.

The fermionic correlator in eq. (3) is

$$(D\cdot\zeta)^{\nu_1\nu_2}(k_1\cdot D)^{\mu_1}k_1^{\mu_2}(D\cdot\zeta)^{\nu_3\nu_4}(k_2\cdot D)^{\mu_3}k_2^{\mu_4}\Psi_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \tag{10}$$

where

$$\Psi_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}=\langle\psi_{\mu_1}(\bar{z}_1)\psi_{\nu_1}(\bar{z}_1)\psi_{\mu_2}(z_1)\psi_{\nu_2}(z_1)\psi_{\mu_3}(\bar{z}_2)\psi_{\nu_3}(\bar{z}_2)\psi_{\mu_4}(z_2)\psi_{\nu_4}(z_2)\rangle\;. \tag{11}$$

To compute this correlator I use the usual Wick contraction valid for even spin-structures

$$\langle \psi^{\mu_1}(z_1)\psi^{\mu_2}(z_2)\rangle \;=\; \frac{(1+(-1)^S)}{2} g^{\mu_1\mu_2} \frac{\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu_{12}|\tau)}{E(z_1,z_2)\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0|\tau)} \tag{12}$$

where $S=(1-2\alpha)(1+2\beta)$ and $\nu_{12}=\frac{1}{2\pi i}\log\frac{z_1}{z_2}$.

Adding the partition function contribution, a generic term of the correlator Ψ looks like

$$\begin{aligned}
&\sum_{\alpha\beta} C^\alpha_\beta k^{1/2}\left(\frac{k^{1/24}}{\eta(\tau)}\right)^4 \frac{(1+(-1)^S)}{2}(-g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\mu_4}g^{\nu_3\nu_4}) \times \\
&\qquad \frac{\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu_{\bar{1}1}|\tau)\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu_{\bar{1}\bar{2}}|\tau)\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu_{12}|\tau)\Theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu_{\bar{2}2}|\tau)}{E(\bar{z}_1,z_1)E(\bar{z}_1,\bar{z}_2)E(z_1,z_2)E(\bar{z}_2,z_2)} = \\
&-\tfrac{1}{2}(-g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\mu_4}g^{\nu_3\nu_4})k^{1/2}\left(\frac{k^{1/24}}{\eta(\tau)}\right)^4 (\eta(\tau))^{12}\prod_{i=1}^4 \omega(z_i)
\end{aligned} \tag{13}$$

where in going to the last line I summed over the spin structures and $\omega(z)=1/z$. Thus one obtains

$$\begin{aligned}
&\sum_{\alpha\beta} C^\alpha_\beta k^{1/2}\left(\frac{k^{1/24}}{\eta(\tau)}\right)^4 \left(\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)\right)^4 \Psi_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \;= \\
&-\tfrac{1}{2}k^{1/2}\left(\frac{k^{1/24}}{\eta(\tau)}\right)^4 (\eta(\tau))^{12}\prod_{i=1}^4 \omega(z_i) \times \\
&\qquad [-g^{\mu_1\mu_2}g^{\nu_1\mu_3}g^{\nu_2\mu_4}g^{\nu_3\nu_4} + 59 \text{ (signed) permutations}]\;.
\end{aligned} \tag{14}$$

Putting together all terms I obtain

$$\begin{aligned}
T^{(Dp)}(k_1,\zeta_1;k_2,\zeta_2) \;=\; & -\frac{1}{2}\left(\frac{\kappa}{\pi}\right)^2 \mathcal{K}(k_1,k_2;\zeta_1,\zeta_2)\int_0^\infty \frac{\mathrm{d}\Im m\tau}{\Im m\tau} \\
& (8\pi^2\alpha'\Im m\tau)^{-(p+1)/2} e^{-(\Delta Y)^2\Im m\tau/4\pi}\int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\bar{z}_1 z_1 \bar{z}_2 z_2} \\
& \prod_{i=1}^2 \exp\left[ik_i\cdot N\cdot Y + \frac{1}{2\pi}k_i\cdot N\cdot \Delta Y\log\left(\frac{z_i}{\bar{z}_i}\right)\right] \times \\
& \exp\left[-\frac{t}{2}\left(g_X(z_1,z_2) - g_X(z_1,\bar{z}_2) - g_X(\bar{z}_1,z_2) + g_X(\bar{z}_1,\bar{z}_2)\right) + \right. \\
& \left. 2q^2\left(g_X(\bar{z}_1,z_1) + g_X(\bar{z}_2,z_2) - g_X(z_1,\bar{z}_2) - g_X(\bar{z}_1,z_2)\right)\right]
\end{aligned} \tag{15}$$

where the kinematical factor $\mathcal{K}(k_1,k_2;\zeta_1,\zeta_2)$ comes entirely from the fermionic correlator and is given by

$$
\begin{aligned}
-\frac{1}{2}\mathcal{K}(k_1,k_2;\zeta_1,\zeta_2) \;=\; & \left(-\frac{t}{2}\right)(2q^2)\left[\mathrm{tr}(\zeta_1\cdot\zeta_2^T)+\mathrm{tr}(D\cdot\zeta_1)\mathrm{tr}(D\cdot\zeta_2)-\mathrm{tr}(\zeta_2\cdot D\cdot\zeta_1\cdot D)\right] \\
& + (2q^2)^2\,\mathrm{tr}(\zeta_1\cdot\zeta_2^T)+\left(-\frac{t}{2}\right)^2\mathrm{tr}(D\cdot\zeta_1)\mathrm{tr}(D\cdot\zeta_2) \\
& + \left(-\frac{t}{2}\right)\left[\mathrm{tr}(D\cdot\zeta_2)\{(k_2\cdot D\cdot\zeta_1\cdot D\cdot k_1)-(k_1\cdot D\cdot\zeta_1\cdot k_2)\}\right. \\
& \qquad + \tfrac{1}{2}(k_2\cdot D\cdot\zeta_2\cdot\zeta_1^T\cdot D\cdot k_1)+\tfrac{1}{2}(k_2\cdot D\cdot\zeta_2^T\cdot\zeta_1\cdot D\cdot k_1) \\
& \qquad \left. -(k_1\cdot D\cdot\zeta_1\cdot D\cdot\zeta_2\cdot D\cdot k_2)+(1\longleftrightarrow 2)\right] \\
& + (2q^2)\left[\mathrm{tr}(D\cdot\zeta_2)(k_2\cdot\zeta_1\cdot k_2)-(k_1\cdot\zeta_2\cdot D\cdot\zeta_1\cdot k_2)\right. \\
& \qquad - \tfrac{1}{2}(k_2\cdot\zeta_1\cdot\zeta_2^T\cdot D\cdot k_2)+\tfrac{1}{2}(k_2\cdot\zeta_1\cdot\zeta_2^T\cdot D\cdot k_1) \\
& \qquad - \tfrac{1}{2}(k_2\cdot\zeta_1^T\cdot\zeta_2\cdot D\cdot k_2)+\tfrac{1}{2}(k_2\cdot\zeta_1^T\cdot\zeta_2\cdot D\cdot k_1) \\
& \qquad \left. +(1\longleftrightarrow 2)\right]\ .
\end{aligned} \tag{16}
$$

4. Comments

A full study of the properties of this amplitude, like its divergencies or possible conformal "anomalies", is left to a future publication. Here I will just make a few comments, starting from the properties of the kinematical factor eq. (16).

The kinematical factor is the same as the "tree level" one, that is the one that it is obtained when a graviton is scattering on a single Dp-brane. In particular, it vanishes for a 9-brane because $D=+1$ and $t=q^2=0$. This is what one expects since in this case there is no scattering at all.

If one sets both polarizations orthogonal to the Dp-brane, the kinematical factor simplifies and for a scattering of a graviton one has $\mathcal{K}(k_1,k_2;\zeta_1,\zeta_2) = (2q^2)^2\mathrm{tr}(\zeta_1\cdot\zeta_2)$ and for an anti-symmetric tensor $\mathcal{K}(k_1,k_2;\zeta_1,\zeta_2) = (2q^2)[4(k_1\cdot\zeta_2\cdot\zeta_1 k_2)+(t-2q^2)\mathrm{tr}(\zeta_1\cdot\zeta_2)]$ whereas for an incoming graviton and an outgoing anti-symmetric tensor the kinematical factor vanishes. [3]

The way in which the amplitude is formulated easily allows to study various properties of this Dp-brane system. For example, one can consider the case where the two Dp-branes are fixed in space. One can then freely choose $Y=0$ and study the behaviour of this

[3] A different model where this contribution does not vanish is studied in ref. [18].

amplitude as a function of ΔY. It will be of particular interest to discuss the limit $\Delta Y \to 0$, that is the limit in which the two Dp-branes coincide in space, in the case of the scattering of Ramond-Ramond states on the Dp-brane [6].

On the other side, as already mentioned, one can integrate over Y and ΔY introducing the corresponding conjugate momenta. Setting $\hat{p} = 0$, one can study the behaviour of this scattering amplitude as a function of the external momenta k_1, k_2, p. For example, one can show that the divergencies which appear when the vertex operators approach the boundary of the cylinder are a result of the expected analytical structure of the amplitude. This contrasts with the case discussed in ref. [4], where it was found a divergence which led to a Weyl anomaly, and ref. [16].

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